

On Optimal Spectrum Access of Cognitive Relay With Finite Packet Buffer

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Abstract—We investigate a cognitive radio system where secondary user (SU) relays primary user (PU) packets using two-phase relaying. SU transmits its own packets with some access probability in relaying phase using time sharing. PU and SU have queues of finite capacity which results in packet loss when the queues are full. Utilizing knowledge of relay queue state, SU aims to maximize its packet throughput while keeping packet loss probability of PU below a threshold. By exploiting structure of the problem, we formulate it as a linear program and find optimal access policy of SU. We also propose low complexity sub-optimal access policies, namely constant probability transmission and step transmission. Numerical results are presented to compare performance of proposed methods and study effect of queue sizes on packet throughput.

Index Terms—Blocking probability, cognitive radio, finite capacity queue, optimal access, relaying

I. INTRODUCTION

In cognitive radio (CR) networks, secondary users (SUs) access spectrum allocated to primary users (PUs) in such a way that given quality-of-service (QoS) requirement of PUs is satisfied. Users store packets arriving from higher layers in queues before transmission over wireless link. Various works have studied SU packet throughput for non-cooperation scenarios where SU's access probability is optimized under queue stability constraint of PU [1]–[3]. Cooperation between SU and PU improves throughput of both users as shown in [4] and [5]. These works consider queues of infinite storing capacity. In practice, queues are of finite size. If a queue is full, new packets cannot be admitted to the queue and are lost. Queueing performance of finite sized SU queue was studied in [6]. In [7]–[11], authors studied relay selection problem for finite buffer-aided relaying systems. These works considered dedicated relay nodes that do not have their own data to transmit. In [12] and [13], authors considered cooperative CR networks with finite sized relay queue and proposed packet admission control assuming that relay queue information is available at SU. However, underlying assumption in these works is that PU queue has infinite buffer length. Also, whole slot is used by the relay either for transmission or for reception. A relaying protocol where packet reception and transmission takes place in the same slot using time sharing, may enable the relay to improve its throughput by transmitting own packets more frequently.

In this paper, we investigate SU throughput in a cooperative CR system where SU relays failed packets of PU using two-phase relaying. SU transmits its own packets in the relaying phase

using time sharing, with some access probability. Furthermore, we consider that both PU and SU have finite capacity queues. SU's finite queue size affects cooperation offered to PU. Thus, queue sizes at both PU and SU impact PU's packet loss. Our aim is to find optimal access policy of SU that maximizes SU packet throughput while satisfying PU's packet loss constraint. Specifically, our contribution is as follows.

- We model PU and relay queues as discrete time Markov chains (DTMC). Using DTMC analysis, we characterize packet loss probability of PU and SU packet throughput.
- We formulate the problem of maximizing SU throughput under PU packet loss constraint, which is non-convex. By exploiting structure of the problem, we transform it into a linear programming (LP) problem over the feasible range of PU packet throughput. We also propose two low-complexity suboptimal access methods that transform original multi-dimensional problem into one dimensional problem.
- Finally, we present numerical results to study effect of queue sizes, path loss and time sharing on SU packet throughput. We also compare the performance with infinite capacity queue system under queue stability constraint.

II. SYSTEM MODEL

As shown in Fig. 1, a PU source \mathcal{P} transmits packets to PU destination \mathcal{D} with assistance of a SU node \mathcal{S} using two-phase relaying as done in [8]. Nodes \mathcal{P} and \mathcal{S} are equipped with packet queue $Q_{\mathcal{P}}$ of capacity $N_{\mathcal{P}}$ and relay queue $Q_{\mathcal{S}}$ of capacity $N_{\mathcal{S}}$ respectively. In a slot of duration T , \mathcal{P} transmits its packet with power $P_{\mathcal{P}}$ for time βT , $\beta \in [0, 1]$. If \mathcal{D} fails to receive the packet, it is admitted to the relay queue at \mathcal{S} , provided that the packet is correctly received at \mathcal{S} and the relay queue is not full. In relaying phase of duration $(1 - \beta)T$, \mathcal{S} relays the PU packet to \mathcal{D} with power $P_{\mathcal{S}}$. With some access probability, SU also transmits its own packets to SU destination \mathcal{R} using time sharing, that is, SU relays PU packet for duration $\alpha(1 - \beta)T$ and transmits its own packet for duration $(1 - \alpha)(1 - \beta)T$, $\alpha \in [0, 1]$. The access probability is p_n when there are n , $0 \leq n \leq N_{\mathcal{S}}$ packets in $Q_{\mathcal{S}}$. If $Q_{\mathcal{S}}$ is empty, whole relaying duration $(1 - \beta)T$ is used to transmit SU packet, with probability $p_0 = 1$.

We assume that all channels are independent block-fading in nature, that is, channel gains remain constant during a slot and vary independently from slot to slot. Channel power gain between source s and destination d is denoted as g_{sd} and is exponentially distributed with mean σ_{sd}^2 , $s, d \in \{\mathcal{P}, \mathcal{S}, \mathcal{D}, \mathcal{R}\}$. The distance between s and d is denoted by r_{sd} and path-loss exponent is denoted by κ . Additive white Gaussian noise (AWGN) at receivers has power $\sigma_{\mathcal{N}}^2$. PU and SU packets have fixed length of \mathcal{B} bits. A packet is assumed to be delivered successfully to

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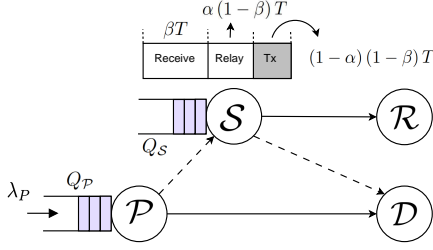


Fig. 1. System model of SU relaying PU packets and transmitting own packets using two-phase relaying and time sharing

intended receiver if instantaneous channel capacity is greater than required transmission rate. Then probability of successful packet transmission is given by [3]

$$\theta_{sd} = \Pr \left[\log_2 \left(1 + \frac{g_{sd} P_s r_{sd}^{-\kappa}}{\sigma_{\mathcal{N}}^2} \right) \geq \frac{\mathcal{B}}{W T_s} \right] = \exp \left(\frac{-\sigma_{\mathcal{N}}^2 \left(2^{\frac{\mathcal{B}}{W T_s}} - 1 \right)}{P_s r_{sd}^{-\kappa} \sigma_{sd}^2} \right), \quad (1)$$

where P_s is transmit power, T_s is transmission duration and W is channel bandwidth. We denote probability of successful packet transmission on $s-d$ link without time sharing by θ_{sd} , $s, d \in \{\mathcal{P}, \mathcal{S}, \mathcal{D}, \mathcal{R}\}$. In case of time sharing, time available for relaying/transmission of PU and SU packets is less than $(1-\beta)T$. We use θ_{SD} and θ_{SR} to denote successful transmission probabilities in case of time sharing¹. As required transmission rate is higher, probabilities of successful packet transmission decrease. Thus, we have $\theta_{SD} < \theta_{SD}$ and $\theta_{SR} < \theta_{SR}$. Successful transmission probabilities on all links are known to the SU [2]–[4].

A. Queue blocking and packet loss

Packet arrival process at PU queue $Q_{\mathcal{P}}$ is Bernoulli with average rate $\lambda_P \in [0, 1]$ packets/slot. A packet is removed from $Q_{\mathcal{P}}$ only when it is received at \mathcal{D} or \mathcal{S} . A PU packet is admitted to the relay queue $Q_{\mathcal{S}}$ when all of the following events are true—1) Packet transmission on $\mathcal{P}-\mathcal{D}$ link fails, 2) PU packet is successfully received at \mathcal{S} , and 3) $Q_{\mathcal{S}}$ is not full. Thus, packet departure rate at $Q_{\mathcal{P}}$, denoted as μ_P , depends on channel between $\mathcal{P}-\mathcal{S}$ and state of $Q_{\mathcal{S}}$. When $Q_{\mathcal{P}}$ is full, new packets cannot be admitted to the queue and are dropped.

PU queue can be modeled as a discrete time Markov chain (DTMC) as shown in Fig 2(a) where states denote number of packets in PU queue. Let w_n , $n = 0, 1, \dots, N_P$ be steady state probability of PU queue being in state n . Also let $\gamma = \frac{\lambda_P(1-\mu_P)}{(1-\lambda_P)\mu_P}$. Then we can write local balance equations for DTMC of $Q_{\mathcal{P}}$ as

$$w_1 = \frac{\gamma}{(1-\mu_P)} w_0, \quad (2)$$

$$w_{n+1} = \gamma w_n, \quad n = 1, 2, \dots, N_P - 1. \quad (3)$$

Noting that $w_n = \gamma^{n-1} w_1$, $n > 1$ and $\sum_{n=0}^{N_P} w_n = 1$, we get probability of $Q_{\mathcal{P}}$ being empty as

$$w_0 = \begin{cases} \frac{(1-\mu_P)(1-\gamma)}{1-\mu_P(1-\gamma)-\gamma^{N_P+1}} & \text{for } \gamma \neq 1 \\ \frac{1-\mu_P}{N_P+1-\mu_P} & \text{for } \gamma = 1. \end{cases}$$

¹Note that notation $\overline{\theta_{sd}}$ only signifies success probability with time sharing and $\overline{\theta_{sd}} \neq 1 - \theta_{sd}$.

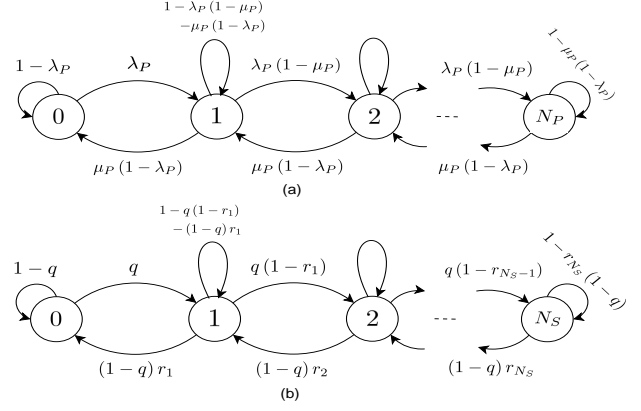


Fig. 2. Discrete time Markov chain (DTMC) model of (a) PU queue $Q_{\mathcal{P}}$ and (b) relay queue $Q_{\mathcal{S}}$

Then probabilities of $Q_{\mathcal{P}}$ being non-empty and $Q_{\mathcal{P}}$ being full are given by $\nu_1 = 1 - w_0$ and $\nu_{N_P} = w_{N_P} = \frac{1}{(1-\mu_P)} \gamma^{N_P} w_0$ respectively. To keep PU packet loss below a limit, probability of $Q_{\mathcal{P}}$ being full should remain below a threshold ϵ , i.e. $\nu_{N_P} \leq \epsilon$. From Fig. 2(a), we observe that, any increase in μ_P would decrease probability of $Q_{\mathcal{P}}$ being full, that is, ν_{N_P} monotonically decreases in μ_P . Thus, for given value of λ_P , we can find $\overline{\mu_P} \in [0, 1]$ such that $\nu_{N_P}(\overline{\mu_P}) = \epsilon$, using Bisection method. Packet loss constraint $\nu_{N_P} \leq \epsilon$ can now be written as $\mu_P \geq \overline{\mu_P}$. SU should choose its access probability in such a way that packet loss constraint of PU is satisfied.

III. OPTIMAL SPECTRUM ACCESS

DTMC of the relay queue $Q_{\mathcal{S}}$ is as shown in Fig. 2(b) where state n denotes number of packets in relay queue at the end of receiving phase. Probability of a PU packet arriving at $Q_{\mathcal{S}}$ is q . When $Q_{\mathcal{S}}$ is in state n , SU transmits its own packets with probability p_n using time sharing. Thus, with probability $(1-p_n)$, PU packet is relayed for duration $(1-\beta)T$ and with probability p_n , PU packet is relayed for duration $\alpha(1-\beta)T$. Then probability of a PU packet departing $Q_{\mathcal{S}}$ in state $n > 0$ is

$$r_n = (1-p_n)\theta_{SD} + p_n\overline{\theta_{SD}} = \theta_{SD} - p_n(\theta_{SD} - \overline{\theta_{SD}}). \quad (4)$$

When PU is present, a packet is received at \mathcal{S} with probability $(1-\theta_{PD})\theta_{PS}$. Thus, we have $q = \nu_1\theta_{PS}(1-\theta_{PD})$. For $1 \leq n < N_S$, state transition from n to $(n+1)$ occurs when packet transmission of a packet in $Q_{\mathcal{S}}$ fails and a new PU packet is received, which happens with probability $q(1-r_n)$. State transition from n to $(n-1)$ occurs when a packet is successfully relayed and no new packet arrives, which happens with probability $(1-q)r_n$. Let π_n , $n = 0, 1, \dots, N_S$ be steady state probability of $Q_{\mathcal{S}}$ being in state n . Then we write local balance equations as

$$\pi_1 = \frac{q}{(1-q)r_1} \pi_0, \quad (5)$$

$$\pi_{n+1} = \frac{q(1-r_n)}{(1-q)r_{n+1}} \pi_n, \quad n = 1, 2, \dots, N_S - 1. \quad (6)$$

For given values of λ_P and μ_P , steady state probabilities of relay queue can be calculated from (5), (6) using

$$\sum_{n=0}^{N_S} \pi_n = 1. \quad (7)$$

At the start of receiving phase, Q_S is full with probability $\pi_{N_S}(1 - r_{N_S})$. As a PU packet is admitted to Q_S only when Q_S is not full, we obtain packet departure rate of PU queue as

$$\mu_P = \theta_{PD} + \theta_{PS}(1 - \theta_{PD})[1 - \pi_{N_S}(1 - r_{N_S})]. \quad (8)$$

A. SU throughput maximization

When there are $n > 0$ packets in Q_S , SU transmits its own packet for duration $(1 - \alpha)(1 - \beta)T$ with probability p_n . If Q_S is empty, whole duration $(1 - \beta)T$ is used to transmit SU packet with probability $p_0 = 1$. Given PU packet arrival rate λ_P , our objective is to maximize SU packet throughput while ensuring that packet loss probability of PU is kept below specified threshold. Thus, the optimization problem is written as

$$\max_{\mathbf{p}, \boldsymbol{\pi}, \mu_P} \quad \mu_S = \theta_{SR}\pi_0 + \overline{\theta_{SR}} \sum_{n=1}^{N_S} \pi_n p_n \quad (9)$$

$$\text{s. t.} \quad \mu_P \geq \overline{\mu_P}, \quad (10)$$

$$0 \leq p_n, \pi_n \leq 1, \quad n = 0, 1, \dots, N_S,$$

$$p_0 = 1$$

$$\mu_P = \theta_{PD} + \theta_{PS}(1 - \theta_{PD})[1 - \pi_{N_S}(1 - r_{N_S})],$$

$$(5), (6), (7),$$

where $\mathbf{p} = [p_0, \dots, p_{N_S}]^T$ and $\boldsymbol{\pi} = [\pi_0, \dots, \pi_{N_S}]^T$. Optimization problem in (9) is non-convex due to product terms of optimization variables π_n and p_n . We transform it into a linear programming (LP) problem by exploiting structure of the problem.

Let $a_n = \pi_n p_n$. Then we have $a_0 = \pi_0$ and $0 \leq a_n \leq \pi_n$, $n = 1, 2, \dots, N_S$. From (7), we have

$$0 \leq \sum_{n=0}^{N_S} a_n \leq 1. \quad (11)$$

Using (4), we can transform balance equations (5) and (6) as given in (12) and (13) on next page.

Similarly, constraint in (8) can be written as

$$(1 - \theta_{SD})\pi_{N_S} + (\theta_{SD} - \overline{\theta_{SD}})a_{N_S} = 1 - \frac{\mu_P - \theta_{PD}}{\theta_{PS}(1 - \theta_{PD})}. \quad (14)$$

Thus, constraints (5), (6), (8) are transformed into constraints (11), (12), (13) and (14) which are affine in π_n and a_n . The optimization problem in (9) is still non-convex in μ_P . However, for a given value of μ_P , the problem becomes a LP problem in variables $\boldsymbol{\pi}$ and $\mathbf{a} = [a_0, a_1, \dots, a_{N_S}]^T$ and is written as

$$\max_{\boldsymbol{\pi}, \mathbf{a}} \quad \theta_{SR}a_0 + \overline{\theta_{SR}} \sum_{n=1}^{N_S} a_n \quad (15)$$

$$\text{s. t.} \quad 0 \leq \pi_n \leq 1, \quad n = 0, 1, \dots, N_S,$$

$$a_0 = \pi_0, \quad 0 \leq a_n \leq \pi_n, \quad n = 1, 2, \dots, N_S,$$

$$(7), (11), (12), (13), (14).$$

From (8) and (10), we see that the feasible values of μ_P are

$$\begin{aligned} \max \{ \overline{\mu_P}, \theta_{PD} + \theta_{PS}\overline{\theta_{SD}}(1 - \theta_{PD}) \} \\ \leq \mu_P \leq \theta_{PD} + \theta_{PS}(1 - \theta_{PD}). \end{aligned} \quad (16)$$

The linear program in (15) is solved over feasible values of μ_P . Value of μ_P that corresponds to the maximum SU packet throughput is chosen. From optimal a_n and π_n , optimal SU access probabilities are found as $p_n = \frac{a_n}{\pi_n}$, $n = 0, 1, \dots, N_S$. We have used CVX package for MATLAB [14] to solve (15) in polynomial complexity.

B. Suboptimal methods

From (4), (5), (6) and (7), we get steady state probabilities for Q_S as

$$\pi_0 = \left[1 + \frac{1}{r_1} \sum_{n=1}^{N_S} \left(\frac{q}{1-q} \right)^n \prod_{m=1}^{n-1} \left(\frac{1-r_m}{r_{m+1}} \right) \right]^{-1}, \quad (17)$$

$$\pi_n = \left[\left(\frac{q}{1-q} \right)^n \frac{1}{r_1} \prod_{m=1}^{n-1} \left(\frac{1-r_m}{r_{m+1}} \right) \right] \pi_0, \quad n > 0. \quad (18)$$

It can be proven that π_0 is monotonically decreasing function of access probability p_n , $n = 1, 2, \dots, N_S$. Also we can prove that π_n , $0 < n \leq N_S$ is a monotonically increasing function of p_m , $m \leq n$ and a monotonically decreasing function of p_m , $m > n$. Intuitively, this can be explained from Fig. 2(b) as follows. As access probability p_m increases, packet departure rate of Q_S decreases. Thus, more packets get queued up in Q_S . Hence, the probability of Q_S having more than m packets increases, while probability of Q_S having packets less than or equal to m decreases. Using this nature, we propose low complexity suboptimal methods that simplify $(N_S + 1)$ dimensional problem in (9) to a one-dimensional problem.

1) *Constant probability transmission (CPT)*: In this method, SU transmits its own packets with a fixed probability p when there are $n > 0$ packets in relay queue. Thus, we have

$$p_n = \begin{cases} 1 & \text{for } n = 0 \\ p & \text{otherwise.} \end{cases} \quad (19)$$

In this case, SU packet throughput is $\mu_S = \theta_{SR}\pi_0 + \overline{\theta_{SR}}p \sum_{n=1}^{N_S} \pi_n$. Using $\sum_{n=1}^{N_S} \pi_n = 1 - \pi_0$, we can write the problem of maximizing μ_S for fixed μ_P as

$$\begin{aligned} \max_{p \in [0, 1]} \quad & \overline{\theta_{SR}}p + \pi_0(\theta_{SR} - \overline{\theta_{SR}}p) \\ \text{s. t.} \quad & (8), (5), (6), (7), (19). \end{aligned}$$

The term $\pi_0(\theta_{SR} - \overline{\theta_{SR}}p)$ is monotonically decreasing in p while term $\overline{\theta_{SR}}p$ is increasing in p . Thus, there exists a unique p that maximizes μ_S . Optimal solution can be found using Interval halving method with complexity $\mathcal{O}(1)$.

2) *Step transmission (ST)*: In this method, SU transmits its own packets using time sharing with probability 1 until length of Q_S reaches a threshold N_{th} . Once it crosses N_{th} , the relaying

$$\theta_{SD} (1 - q) \pi_1 - q \pi_0 = (\theta_{SD} - \overline{\theta_{SD}}) (1 - q) a_1, \quad (12)$$

$$\theta_{SD} (1 - q) \pi_{n+1} - q (1 - \theta_{SD}) \pi_n = (\theta_{SD} - \overline{\theta_{SD}}) (1 - q) a_{n+1} + q (\theta_{SD} - \overline{\theta_{SD}}) a_n, \quad n = 1, \dots, N_S - 1. \quad (13)$$

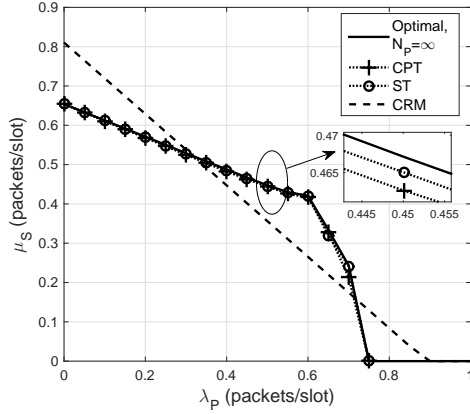


Fig. 3. SU packet throughput μ_S versus PU packet arrival rate λ_P for $N_S = 10$ and $N_P \rightarrow \infty$.

phase duration of $(1 - \beta)T$ is used only to relay PU packets. Thus, we have

$$p_n = \begin{cases} 1 & \text{for } n \leq N_{th} \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

In this case, the objective is

$$\begin{aligned} \max_{N_{th} \in \{0, 1, \dots, N_S\}} \quad & \theta_{SR} \pi_0 + \overline{\theta_{SR}} \sum_{n=1}^{N_{th}} \pi_n \\ \text{s. t.} \quad & (5), (6), (7), (8), (20). \end{aligned}$$

With increasing N_{th} , π_0 decreases while number of terms in summation increase. If value of $\overline{\theta_{SR}}$ is very low compared to θ_{SR} , decrease in π_0 is significant and μ_S initially decreases. But as π_0 approaches zero, μ_S increases due to increasing value of $\overline{\theta_{SR}} \sum_{n=1}^{N_{th}} \pi_n$. For high value of $\overline{\theta_{SR}}$, μ_S increases with increasing N_{th} . Throughput drops to zero when π_{N_S} increases to such a value that constraint (4) cannot be satisfied. Value of N_{th} that maximizes μ_S can be found by linear search with complexity $\mathcal{O}(N_S)$.

Suboptimal methods run over all feasible values of μ_P given in (16) and the value that corresponds to maximum SU packet throughput is chosen.

IV. NUMERICAL RESULTS AND DISCUSSION

Parameter values used to plot results are as follows. Transmit power is $P_P = P_S = 0.1$ W. Frame duration is $T = 100$ ms. Time sharing factors are $\beta = \alpha = 0.5$ unless stated otherwise. All channels have average channel gain $\sigma_{sd}^2 = -10$ dB, $s, d \in \{\mathcal{P}, \mathcal{S}, \mathcal{D}, \mathcal{R}\}$. Noise power is $\sigma_N^2 = 10^{-5}$ W. We take $\mathcal{B}/W = 3 \times 10^{-3}$ bits/Hz. We consider $r_{PS} = r_{SD} = r_{SR} = 100$ m. Path loss exponent is $\kappa = 2$. Packet loss probability threshold is $\epsilon = 0.01$.

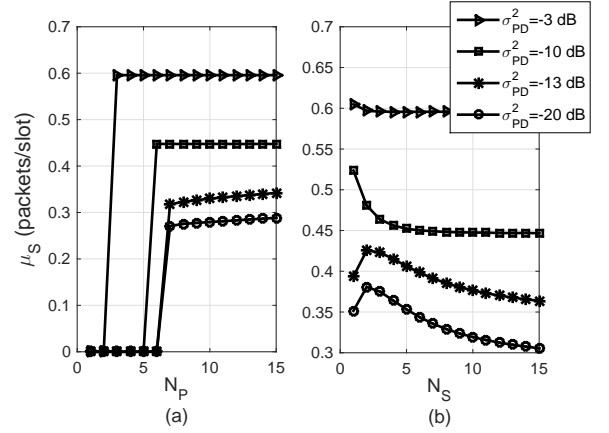


Fig. 4. Effect of queue sizes on SU packet throughput μ_S for (a) $N_S = 10$ (b) $N_P = 100$ and $\lambda_P = 0.5$

3) *Throughput region*: Fig. 3 plots throughput region of proposed cooperation model. As λ_P increases, higher μ_P is required to satisfy PU packet loss constraint. To support high μ_P , SU lowers its access probability. Thus, μ_S decreases with increase in λ_P . As λ_P increases further, constraint (10) becomes infeasible, at which point μ_S drops to zero. We also see that performance of constant probability transmission (CPT) method and step transmission (ST) method is close to that of optimal method. Hence, the suboptimal methods are good low-complexity alternatives to the optimal method.

As a baseline for comparison, we also plot throughput region of cooperative relaying method (CRM) in [13]. In CRM, PU utilizes whole undivided frame duration T for transmission/reception and optimizes SU access probability under PU queue stability constraint. In contrast, two-phase relaying model dedicates βT duration for reception in each slot. Thus, in CRM, probabilities of successful transmission on $\mathcal{P} - \mathcal{D}$ and $\mathcal{S} - \mathcal{R}$ links are higher, resulting in better performance of CRM at low and high values of λ_P . But in mid-range of λ_P , two-phase relaying benefits by gaining time to transmit own packets as SU relays PU packets in the same slot.

4) *Effect of queue sizes*: Fig. 4(a) plots SU packet throughput μ_S against PU queue capacity N_P for different values of $\mathcal{P} - \mathcal{D}$ channel gains. Low values of N_P cannot satisfy packet loss constraint in (10). Packet throughput achieved in such infeasible cases is zero. Increasing N_P decreases $\overline{\mu_P}$ which is the minimum PU departure rate required to satisfy packet loss constraint. This allows SU to transmit its own packets with higher access probabilities. Thus, μ_S increases with increase in N_P . As N_P increases further, decrease in $\overline{\mu_P}$ is insignificant. Access probabilities of SU become constant and in turn μ_S becomes constant. For high value of σ_{PD}^2 , PU packet arrival rate at Q_S is less which allows higher SU access probabilities. Thus, μ_S increases as σ_{PD}^2 increases.

Fig. 4(b) shows an interesting tradeoff involving relay queue

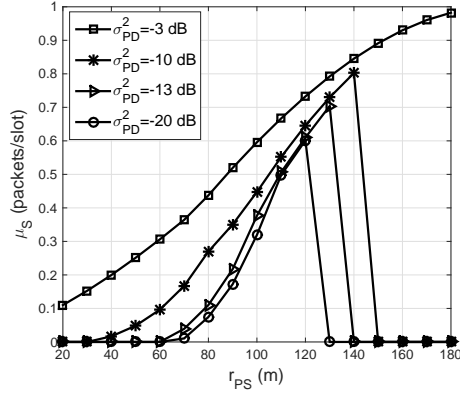


Fig. 5. Effect of distance between PU source and relay r_{PS} on SU packet throughput μ_S for $N_P = 100$, $N_S = 10$, $\lambda_P = 0.5$ and $r_{PD} = 200$ m.

capacity N_S . Increase in N_S allows SU to transmit its own packets with higher probability. Also, from (17), we observe that increase in N_S decreases probability of relay queue being empty π_0 . For high values of σ_{PD}^2 , PU packet arrival rate at Q_S is less. In this case, decrease in π_0 (and subsequent decrease in $\theta_{SR}\pi_0$) is significant compared to increase in μ_S due to higher access probability. Thus, μ_S decreases with increase in N_S . For low values of σ_{PD}^2 , PU packet arrival rate at Q_S is more. In this case, increase in SU throughput due to higher access probability is significant. But as N_S increases further, π_0 approaches zero and $\sum_{n=1}^{N_S} \pi_n p_n$ decreases. Thus, with increasing N_S , μ_S initially increases and then gradually decreases.

5) *Effect of distance*: Fig. 5 plots SU throughput against distance between PU source and SU source r_{PS} . Here, we assume that \mathcal{D} and \mathcal{R} are in close vicinity and lie on the line connecting \mathcal{P} and \mathcal{S} . Then for given r_{PD} , we have $r_{SD} = r_{SR} = r_{PD} - r_{PS}$. When $\mathcal{P}-\mathcal{D}$ channel is weak, PU packet arrival rate at SU is high. Thus, probability of Q_S being full is high. As SU moves away from PU source, θ_{PS} decreases, while θ_{SD} , θ_{SR} , θ_{SD} and θ_{SR} increase. This increases SU throughput. As r_{PS} increases further, μ_P decreases to such a point that queue blocking constraint cannot be satisfied for given λ_P . In this infeasible region, μ_S is zero. When $\mathcal{P}-\mathcal{D}$ channel is strong, decrease in μ_P due to increasing r_{PS} is insignificant. Thus, SU throughput μ_S keeps increasing as \mathcal{S} moves closer to \mathcal{R} .

6) *Effect of time sharing*: Fig. 6 plots μ_S against time sharing factors β and α . When β is low, values of θ_{PD} and θ_{PS} are low. This results in lower value of μ_P that cannot support given λ_P . As β increases, PU packet departure rate increases. This allows SU to transmit its own packets with non-zero probability. Thus, μ_S increases. As β increases further, less time is available for SU to transmit its own packets which decreases θ_{SR} . Thus, μ_S decreases at high value of β . As α increases, θ_{SD} increases while θ_{SR} decreases. Increase in departure rate of PU packets from Q_S allows SU to transmit its own packet with higher access probability. Thus, μ_S increases with increasing α . But as α increases further, decrease in θ_{SR} becomes dominant, in turn decreasing μ_S . This indicates that there is scope to improve μ_S by optimizing β and α . However, the problem is difficult to solve

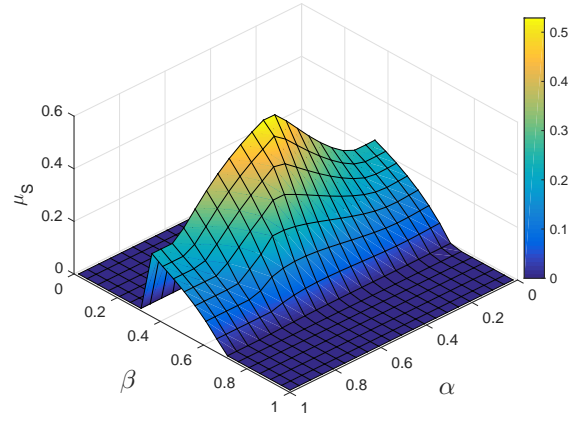


Fig. 6. Effect of time sharing factors β and α on SU packet throughput μ_S for $\lambda_P = 0.5$, $N_S = 10$, $N_P = 50$ and $\sigma_{PD}^2 = -20$ dB.

as objective in (9) is non-convex in β and α .

V. CONCLUSION

We studied a CR system where SU employs two-phase relaying to relay failed PU packets. Both users have packet queues of finite capacity which results in packet loss. We proposed optimal as well as suboptimal access methods for SU to maximize its packet throughput under packet loss constraint of PU. We observed that two-phase relaying model performs better than cooperation model without time sharing for mid-range values of PU packet arrival rate. Suboptimal methods have negligible loss in the performance and are good low complexity alternatives to the optimal method. Furthermore, results revealed that as relay queue size increases, SU throughput improves initially but then decreases. PU queue size limits maximum supported PU packet arrival rate.

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